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Problem of Optimum Position Intersection Between Two Cylindrical Shells of Revolution

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Summary: *This paper investigates the problem of temperature extrema in a thin cylindrical shell under local axially symmetrical heating when this problem is used to calculate the optimum parameters of 3D shell structures. The relevant variation problem is defined and the equations which determine the extremum thermal load are derived. A mathematical model is provided to study thermal stress at the intersection line of two connected shells.*

Key words: *Local heating, temperature field, thermotension, extreme conditions, cylindrical cover, temperature tension, elastic energy of a cover, functional.*

1. INTRODUCTION

In general, the shell-type structures in power machine industry such as pressure vessels, tanks, reservoirs and other items are produced with welded connections including nozzles, throats, fasteners, etc. Such connections have intersection lines of various geometry which varies in plan view from a circle to ellipse or other second-order curve. It is an important task for designer to determine the best position of the welded item with sufficient thermoelastic capacity.

Let's consider the problem of intersection between two cylindrical shells of revolution. For the given axially symmetric loads and the resulting temperature field we shall take a group of shells which satisfy certain constraints and find the shell which retains thermal elastic properties.

2. PROBLEM OF INTERSECTION BETWEEN TWO CYLINDRICAL SHELLS OF REVOLUTION

The problem conditions are defined as follows. A free thin isotropic cylindrical shell $S_1(R)$ (hereinafter called a bearing shell) is connected with a thin cylinder $S_2(r)$ of radius r, and the thin cylinder intersects the main cylindrical shell at angle φ to the outside normal line n in point 0, where $R >> r$. The area of connection suffers local heating with the maximum temperature T_1. The boundary conditions are of the third type when both cylindrical shells satisfy the following equations:

$$\frac{\partial t^{1,2}}{\partial \gamma} = 0\,;\ T^{1,2}_{(+h)} = T^{1,2}_{(-h)} = \text{const.}$$

Indices 1, 2 correspond to the main and connected cylindrical shells while indices ($+h$) and ($-h$) relate to the internal and external surfaces of these shells respectively.

It is required to determine the maximum allowable equivalent stress caused by thermal factor at the intersection line between two members under thermoelastic conditions.

We will study alternatives where the connected thin cylinder is positioned by clockwise rotation of its longitudinal axis in point 0. Please refer to Figure 1 for the details.

Let's write a matching condition for intersection of two cylindrical shells of revolution. If S_1 is a selected area of the main shell and S_2 is a selected area of the connected cylinder then S_2 crosses S_1 along the curve m.

$S_2 \cap S_1 = m;\ M \in m;\ M \in S_1;\ M \in S_2.$

The required approximate solution may be found after solving a model problem of local extremum in the temperature field of thin shells under specified conditions of heating. The problems of this type are defined and solved in a number of papers [1-5].

The minimum of function of shell elastic energy is considered to be estimation criterion of optimality. The relevant variation problem may be defined as on Figure 1.

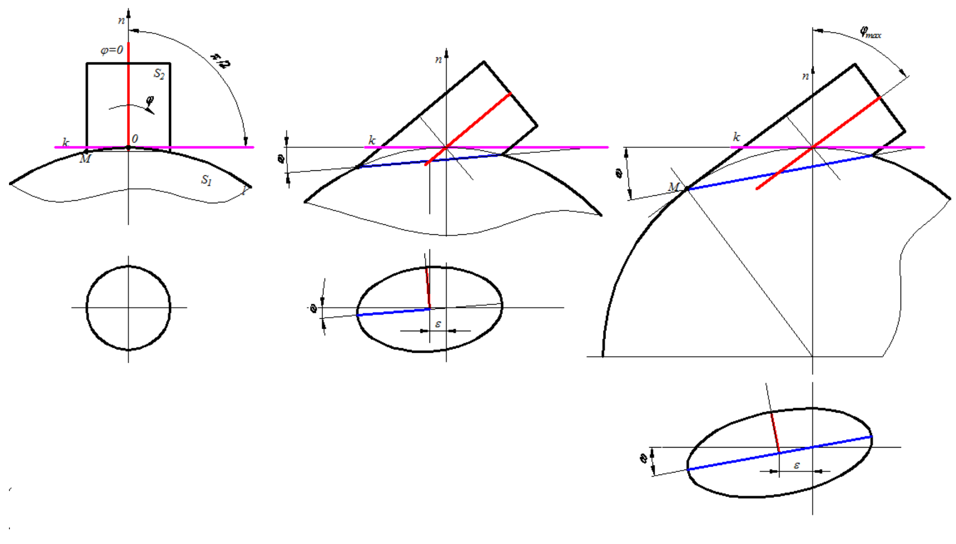

Figure 1. Relative position of two intersecting thin cylindrical shells of revolution

It is required to find the extremum of function of shell elastic energy on a set of displacement functions u, v, w and thermal force and momentum functions T_1, T_2, which satisfy the equation system, attachment conditions of the edge cross section and specified additional restraint conditions.

In this case it would be useful to consider a flat problem of cylindrical shell when the effect of temperature field is found in plane section, Figure 2.

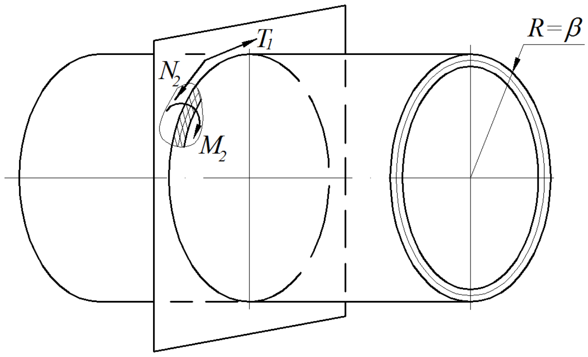

Figure 2. Plane problem of cylindrical shell

The temperature field may be described by coordinate parameter β and there is function T_1 (β), where $R = \beta$; buckling function is $w = f(R)$, $T_2 = 0$.

The boundary conditions are defined as follows:

$$\frac{\partial t}{\partial \gamma} = 0; \quad T_{(+h)} = T_{(-h)} = \text{const}.$$

Influence of forces in the cylindrical shell is determined by the equations:

$$N_2 = -\frac{D_1\alpha_t(1+\nu)}{\pi R^2}\left(1+\frac{h^2}{3R^2}\right)^{-1}\int_0^{2\pi} T_1\cos(\beta-\beta_0)\,d\beta_0;$$
$$M_1 = 0; \quad M_2 = -R\,N_2, \tag{1}$$

where D_1 is bending stiffness; $D_1 = \frac{2Eh^3}{3(1-\nu^2)}$; β is a coordinate of mid-surface of the shell; α_t is a coefficient of linear thermal expansion; T_1 is an average characteristic of temperature field, $T_1 = \frac{1}{2h}\int_{-h}^{h} t\partial\gamma$; E is modulus of elasticity; ν is Poison's ratio; $2h$ is a shell thickness.

Let's write the equation of temperature function which is associated with buckling function:

$$\frac{\partial^2 w_0}{\partial \beta^2} + w_0 - \alpha_t(1+\nu)T + \frac{\alpha_t\nu}{2\pi}\int_0^{2\pi} T d\beta_0 + \frac{\alpha_t(1+\nu)}{\pi}\int_0^{2\pi} T\cos(\beta-\beta_0)\,d\beta_0 = 0;$$
$$\int_0^{2\pi}(w_0-\alpha_t T)\,d\beta = 0. \tag{2}$$

A variation problem is defined as follows. It is required to find the extremum of function of shell elastic energy on a set of functions w_0 (β) and T (β), each function satisfies the equations (2) and additional constraints for buckling function in the cross sections:

$\beta = \beta_i$ ($i = 1, 2, \ldots, n$)

such as:

$w_0\ (\beta_i) = a_{1i},\qquad \int_0^{2\pi} w_0\left(\beta\right)d\beta = a_{0i}$

and ensures that the following functions are stationary:

$$K_1 = \int_0^{2\pi} w_0\delta\left(\beta-\beta_i\right)d\beta_0\ ;$$

$$K_2 = \int_0^{2\pi} \theta\left(\beta_i-\beta\right)w_0 d\beta_0\ ;$$

$K_1 = a_{1i},\quad K_2 = a_{0i}.$

Let's use a particular solution of thermal conductivity problem of cylindrical shell with boundary conditions $\frac{\partial t}{\partial \gamma}=0$. In this case the following family of the extremum temperature fields are derived as a solution of the defined variation problem:

$$T_1 = A\cos\beta + B\sin\beta + \sum_{i=1}^{n}\left\{\gamma_{0i}\left[1-\cos\left(\beta-\beta_i\right)+\gamma_{1i}\sin\left(\beta_i-\beta\right)\right]\right\}\theta\left(\beta_i+\beta\right)+ + \frac{1}{2\pi}\int_0^{2\pi} T_1 d\beta_0\ . \qquad (3)$$

where $\xi = \cos\ (\beta - \beta_i)$; $\zeta = \sin\ (\beta_i - \beta)$.

Lagrange multipliers satisfy the conditions:

$$\sum_{i=1}^{n}\left[\gamma_{0i}\left(\beta_i-\sin\beta_i\right)+\gamma_{1i}\left(1-\cos\beta_1\right)\right]=0\ ;$$

$$\sum_{i=1}^{n}\left[\gamma_{0i}\left(1-\cos\beta_i\right)+\gamma_{1i}\sin\beta_i\right]=0\ .$$

Let's take the general solution of the variation problem and extract the extremum temperature field, whose first derivative and the field itself is continuous in β and which satisfies the following conditions on the surface (Figure 3):

$T\ (\pi) = T_1,\ \ T_0\ (-\ \beta_1) = T_0\ (+\ \beta_1) = 0,\ T^{'}\ (\pi) = 0;$
$2\pi - \beta_1 \le \beta_1\ (0) \le 0 + \beta_1;\ 0 \le (+\ \beta_1) \le \pi;\ \pi \le (-\ \beta_1) \le 2\pi.$ (4)

This particular solution looks like [2]:

$$T_1 = T_0\,\frac{\cos\beta_1-\cos\beta}{1+\cos\beta}\ . \qquad (5)$$

We set:

$$A = \frac{\cos\beta_1\left(0\right)-\cos\left(+\beta_1\right)}{1+\cos\left(+\beta_1\right)}\ . \qquad (6)$$

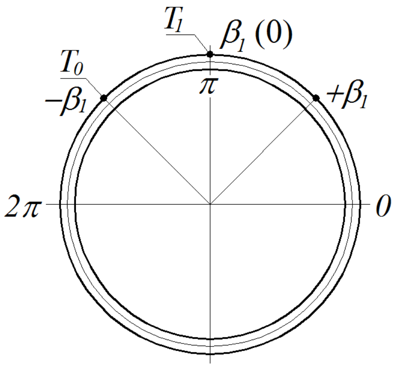

Figure 3. Surface conditions applied for the solution of the problem

Characteristic parameter A is a coordinate of the unit function of temperature field $t_0 = f\,(A)$, which acts in the plane section of cylindrical shell and achieves its maximum in point β_1 (0) and which belongs to the family of extremum temperature fields [3].

Now we consider an infinite cylindrical shell with loose ends which is subjected to influence of variable temperature field acting in the plane tilted to the shell longitudinal axis. We assume that this is a plane problem of cylindrical shell, see Figure 4. The shell has the same parameters as taken before.

The temperature field may be described by coordinate parameter β and there is function $T\,(\beta)$, where β is a coordinate of the ellipse generated by the plane section tilted at angle θ to the longitudinal axis of cylindrical shell. The boundary conditions are the same as defined for the previous problem.

Let's take the earlier derived solution (5, 6) as a particular solution of the current problem where the plane section is tilted at angle $\theta = 0$ and the radius of the cylindrical shell is a radius of curvature $\rho\,(\beta)$ of the new generated figure. Based on geometrical relationship (Figure 4) it follows that:

- ρ achieves the maximum value in the top pole of ellipse where $\rho = R\,/\cos\,\theta$;
- ρ has the minimum value at the small axis of the ellipse where $\rho = R$.

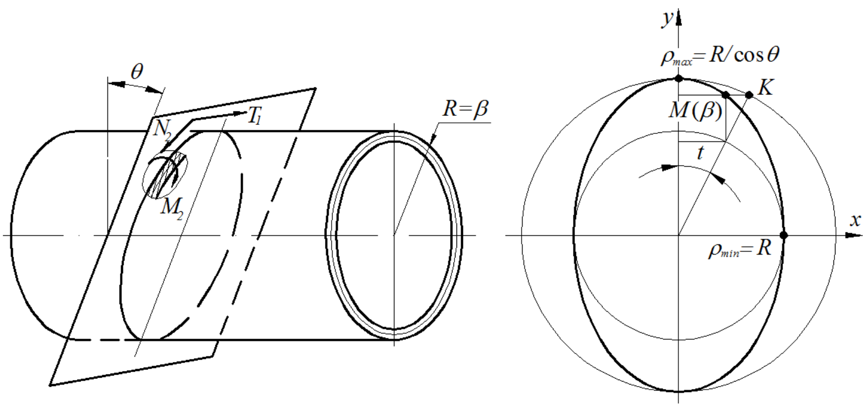

Figure 4. Forces which act in the plane tilted at angle θ to the longitudinal axis of cylindrical shell

With parametric equation of ellipse, we get:

$\rho\,(x) = R \sin t$; $\rho\,(y) = (R\,/\cos\,\theta) \cos t$; $0 \le t \le 2\pi$.

Now we define non-dimensional radius of curvature:

$$\rho = \frac{\sqrt{\cos^2 t + \sin^2 t \cos^2 \theta}}{\cos \theta}.$$

It is easy to check that at $\theta = 0$, when the plane is perpendicular to the longitudinal axis of the shell, radius of curvature ρ is equal to one (the curve shape is a circumference).

The extremum temperature field which satisfies the same conditions at the surface (4), Figure 5, may be defined by the equation:

$$T_1 = T_0 \left(\frac{\sqrt{\cos^2 t + \sin^2 t \cos^2 \theta}}{\cos \theta} \right) \frac{\cos \beta_1 - \cos \beta}{1 + \cos \beta}. \tag{7}$$

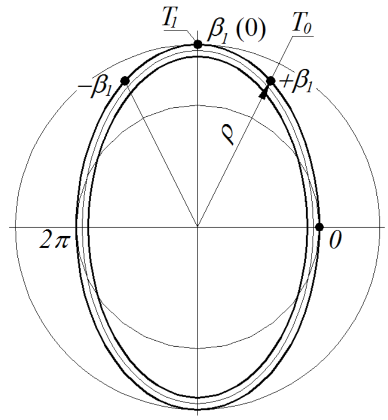

Figure 5. Conditions at the surface

We set:

$$A_1 = \left(\frac{\sqrt{\cos^2 t + \sin^2 t \cos^2 \theta}}{\cos \theta} \right) \frac{\cos \beta - \cos \beta_1}{1 + \cos \beta_1}. \tag{8}$$

Characteristic parameter A_1 is a coordinate of the unit function of temperature field $t = f(A_1)$, which acts in the tilted plane section of cylindrical shell and achieves its maximum in point β_1 (0) and which belongs to the family of extremum temperature fields (7).

Find below the curves of unit function of temperature field versus coordinate parameters A and A_1 when the plane section is tilted at various angle $0° \leq \theta \leq 80°$to the longitudinal axis of the shell, Figure 6. These calculations are made with LABVIEW software with the help of operator prepared for specified values of parameter t within the range $0 \leq t \leq 90°$ at a step of 10°.

It is obvious that the level of extremum temperature differs depending on the different shapes of shell cross section which is cut by the plane subjected to the local heat load. The minimum level corresponds to the shell cross section which has a shape of circumference. All other cross sections in the shape of ellipse produce higher level of extremum temperature. The greater the angle between the tilted plane section and the shell longitudinal axis, the higher maximum temperature when thermoelastic properties are retained.

If we solve an inverse problem and take the earlier obtained extremum temperature as the limit values for transient processes from elastic conditions to elastoplastic conditions, we may define the maximum allowable equivalent stress by the formula:

$$\sigma_{\max} = \frac{E\alpha T_{\max}}{2(1-\nu)},$$

where $T_{\max}$ corresponds to equation (7).

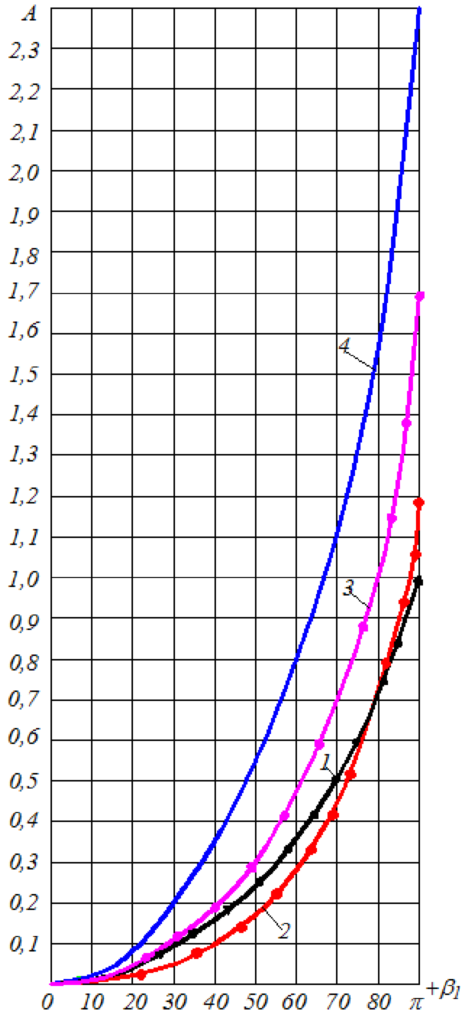

Figure 6.Unit function of temperature field which acts in particular plane section tilted to the axis of cylindrical shell: curve 1 corresponds to the cross section perpendicular to the axis of cylindrical shell, angle $\theta = 0$;curve 2 corresponds to the cross section cut by the plane which is tilted at angle $\theta = 45°$; curve 3 corresponds to the cross section cut by the plane which is tilted at angle $\theta = 70°$; curve 4 corresponds to the cross section cut by the plane which is tilted at angle $\theta = 80°$

3. CONCLUSION

The proposed model problem is used to find the best solutions in design of intersection between two cylindrical shells, it is preferable to combine the optimum position of the connected member and optimum manufacturing process.

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